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18MAT21

Second Semester B.E. Degree Examination, Aug./Sept.2020
Advanced Calculus and Numerical Methods

Time: 3 hrs. Max. Marks:100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

1 a. Find the angle between the surfaces $x^2 + y^2 - z^2 = 4$ and $z = x^2 + y^2 - 13$ at (2, 1, 2). **(06 Marks)**

b. If $\vec{F} = \nabla(xy^3z^2)$, find div $\vec{F}$ and curl $\vec{F}$ at (1,–1, 1). **(07 Marks)**

c. Find the value of the constant a such that the vector field

$$\vec{F} = (axy - z^3)\hat{i} + (a-2)x^2 j + (1-a)xz^2 k$$

is irrotational and hence find a scalar function ϕ such that $\vec{F} = \nabla\phi$. **(07 Marks)**

OR

2 a. If $\vec{F} = (3x^2 + 6y)i - 14yzj + 20xz^2k$, evaluate $\int_C \vec{F}\cdot d\vec{r}$ from (0, 0, 0) to (1, 1, 1) along the curve given by $x = t$, $y = t^2$ and $z = t^3$. **(06 Marks)**

b. Use Green's theorem to find the area between the parabolas $x^2 = 4y$ and $y^2 = 4x$. **(07 Marks)**

c. If $\vec{F} = 2xyi + yz^2j + xzk$ and s is the rectangular parallelopiped bounded by x = 0, y = 0, z = 0 and x = 2, y = 1, z = 3. Find the flux across S. **(07 Marks)**

Module-2

3 a. Solve $(D^2 + 3D + 2)y = 4\cos^2 x$. **(06 Marks)**

b. Solve $(D^2 + 1)y = \sec x \tan x$, by the method of variation of parameter. **(07 Marks)**

c. Solve $x^2y'' + xy' + 9 = 3x^2 + \sin(3\log x)$. **(07 Marks)**

OR

4 a. Solve $y'' + 2y' + y = 2x + x^2$. **(06 Marks)**

b. Solve $(2x + 1)^2y'' - 6(2x + 1)y' + 16y = 8(2x + 1)^2$. **(07 Marks)**

c. The current i and the charge q in a series circuit containing on inductance L, capacitance C, emf E satisfy the differential equation : $L\frac{di}{dt} + \frac{q}{c} = E;\ i = \frac{dq}{dt}$. Express q and i interms of t, given that L, C, E are constants and the value of i, q are both zero initially. **(07 Marks)**

Module-3

5 a. Form the partial differential equation by eliminating the arbitrary function from $\phi(xy + z^2, x + y + z) = 0$. **(06 Marks)**

b. Solve $\frac{\partial^2 z}{\partial x \partial y} = \sin x \sin y$ for which $\frac{\partial z}{\partial y} = -2\sin y$ when x = 0 and z = 0 if $y = (2n+1)\frac{\pi}{2}$. **(07 Marks)**

c. Derive one dimensional wave equation in the standard form $\frac{\partial^2 u}{\partial t^2} = c^2\frac{\partial^2 u}{\partial x^2}$. **(07 Marks)**

OR

6 a. Form the partial differential equation by eliminating the arbitrary function form $f\left(\frac{xy}{z}, z\right) = 0$. **(06 Marks)**

b. Solve $\frac{\partial^2 z}{\partial y^2} = z$, given that when $y = 0$, $z = e^x$ and $\frac{\partial z}{\partial y} = e^{-x}$. **(07 Marks)**

c. Find all possible solutions of one dimensional heat equation $\frac{\partial u}{\partial t} = C^2 \frac{\partial^2 u}{\partial x^2}$ using the method of separation of variables. **(07 Marks)**

Module-4

7 a. Test for convergence of the series $\sum_{n=1}^{\infty} \frac{(n!)^2}{(2n)!} x^n, (x > 0)$. **(06 Marks)**

b. Solve the Bessel's differential equation leading to $J_n(x)$. **(07 Marks)**

c. Express $f(x) = x^4 + 3x^3 - x^2 + 5x - 2$ interms of Legendre's polynomials. **(07 Marks)**

OR

8 a. Test for convergence of the series $\sum_{n=1}^{\infty} \frac{n^2}{2^n}$. **(06 Marks)**

b. If α and β are two distinct roots fo $J_n(x) = 0$. Prove that $\int_0^1 x J_n(\alpha x) J_n(\beta x) dx = 0$. If $\alpha \neq \beta$. **(07 Marks)**

c. Express $f(x) = x^3 + 2x^2 - x - 3$ interms of Legendre's polynomials. **(07 Marks)**

Module-5

9 a. Find the real root of the equation : $x^3 - 2x - 5 = 0$ using Regula Falsi method, correct to three decimal places. **(06 Marks)**

b. Use Lagrange's formula, find the interpolating polynomial that approximates the function described by the following data :

x	0	1	2	5
f(x)	2	3	12	147

c. Evaluate $\int_0^1 \frac{x\,dx}{1+x^2}$ by Weddle's rule, taking seven ordinates and hence find $\log e^2$.

OR

10 a. Find the real root of the equation $xe^x - 2 = 0$ using Newton – Raphson method correct to three decimal places.

b. Use Newton's divided difference formula to find f(4) given the data :

x	0	2	3	6
f(x)	–4	2	14	158

c. Use Simpson's $\frac{3}{8}^{th}$ rule to evaluate $\int_1^4 e^{1/x} dx$.

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